Transformations of Rationals and others

Period _____Date ____

- I. As we've learned earlier, $y = x^2$ can be transformed into $f(x) = a \cdot (x h)^2 + k$.
 - 1. What effect does a have on the graph of $y = x^2$?

 Vertically stretches (or compresses if abs(a) < 1) the entire graph by a factor of a.
 - 2. What effect does h have on the graph of $y = x^2$?

 Shifts the entire graph h units to the <u>left</u> (if positive) or <u>right</u> (if negative).
 - 3. What effect does k have on the graph of $y = x^2$?

 Shifts the entire graph k units up (if positive) or down (if negative).

II. You also know what $y = x^3$ looks like now. So consider the graph $g(x) = a \cdot (x - h)^3 + k$.

- 1. What effect does a have on the graph of $y = x^3$?

 Vertically stretches (or compresses if abs(a) < 1) the entire graph by a factor of a.
- 2. What effect does h have on the graph of $y = x^3$?

 Shifts the entire graph h units to the <u>left</u> (if positive) or <u>right</u> (if negative).
- 3. What effect does k have on the graph of $y = x^3$?

 Shifts the entire graph k units up (if positive) or down (if negative).

III. The parent graph of a rational function is $y = \frac{1}{x}$. So consider $h(x) = k + \frac{a}{x - h}$.

1. What effect does **a** have on the graph of $y = \frac{1}{x}$?

Vertically stretches (or compresses if abs(a) < 1) the entire graph by a factor of a.

2. What effect does h have on the graph of $y = \frac{1}{x}$?

Shifts the entire graph h units to the <u>left</u> (if positive) or <u>right</u> (if negative).

3. What effect does k have on the graph of $y = \frac{1}{x}$?

Shifts the entire graph k units \underline{up} (if positive) or \underline{down} (if negative).

IV. Look at all the cases above, and then describe the locations of the variables in relation to the parent graph.

1. Where is a located? It's the coefficient in front of the function.

What does a do?

Vertically stretches (or compresses if abs(a) < 1) the entire graph by a factor of a. What happens when a < 0? It reflects the graph over the x-axis.

2. Where is h located?

Added or subtracted directly to x.

What does h do?

Shifts the entire graph h units to the <u>left</u> (if positive) or <u>right</u> (if negative).

3. Where is k located?

Added or subtracted to the function.

What does k do?

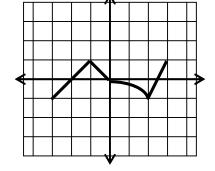
Shifts the entire graph k units <u>up</u> (if positive) or down (if negative).

V. Consider the sketch of f(x) to the right. Answer questions about f(x) and sketch

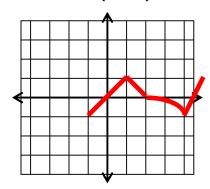
each of the following transformations of f(x).

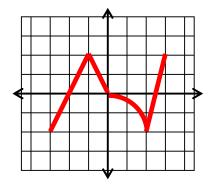
1. f(1) = <u>0.25</u> 2. f(2) = <u>-1</u>

- 3. f(0) = <u>0</u> 4. f(-1) = <u>1</u>

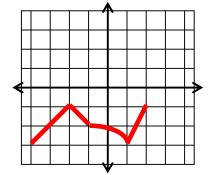


5. Sketch f(x-2) 6. Sketch 2f(x)

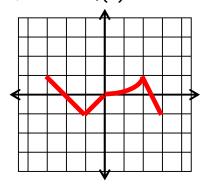




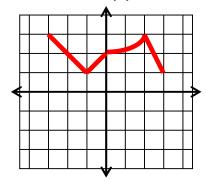
7. Sketch f(x + 1) - 2



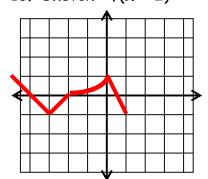
8. Sketch -f(x)



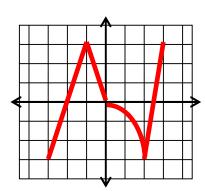
9. Sketch -f(x) + 2



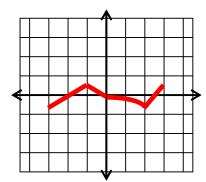
10. Sketch -f(x + 2)



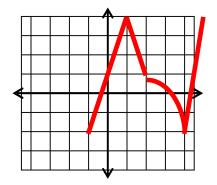
11. Sketch 3f(x)



12. Sketch $(\frac{1}{2})f(x)$



13. Sketch 1 + 3f(x - 2)



VI. For each of the following: a) **Evaluate** f(2), b) Describe the **transformations** of y = 1/x that have been done to this function, c) Use this information to **sketch** the graph, d) Find the **domain** of the function, e) Find the **range** of the function, and f) Write **equations** of all the **asymptotes**.

Problem:	$f(x) = \frac{2}{x-1}$	$g(x) = \frac{-3}{x} + 4$	$h(x) = \frac{1}{2(x+3)} - 1$
Evaluate at x = 2	f(2) = 2	g(2) = 5/2	h(2) = -9/10
Transformations	Shift 1 unit to the right; Vertical stretch by factor of 2.	Shift 4 units up; Raflect over x-axis; Vertical stretch by 3.	Shift 1 unit down; Shift 3 unitd to the left; Stretch by 1/2.
Sketch the graph			
Domain	x ≠ 1	x ≠ 0	x ≠ -3
Range	y ≠ 0	y ≠ 4	y ≠ -1
Asymptotes	x = 1 and $y = 0$	x = 0 and $y = 4$	x = -2 and $y = -1$

VII. For each of the following: a) **Evaluate** f(-3), b) **Divide** this rational function to write its **quotient** in transformation form, c) Describe the **transformations** of y = 1/x that have been done to this function, d) Use this information to **sketch** the graph, e) Find the **domain** of the function, f) Find the **range** of the function, and g) Write **equations** of all the **asymptotes**.

Problem:	$f(x) = \frac{x-2}{x+1}$	$g(x)=\frac{3x+5}{x-2}$	$h(x) = \frac{x-4}{2x-5}$
Evaluate x = -3	f(-3) = 2.5	g(-3) = .8	h(-3) = 7/11
Quotient	$f(x) = 1 - \frac{3}{x+1}$ or $-\frac{3}{x+1} + 1$	$g(x) = \frac{11}{x-2} + 3$	h(x) = $-\frac{3/2}{2x-5} + \frac{1}{2} = -\frac{3}{2(2x-5)} + \frac{1}{2}$
Transformations	Left 1, up 1, Reflects across the x axis	Right 2, up 3	Right 5/2, up $\frac{1}{2}$, reflect across x axis
Sketch the graph	(Graph in graphing calculator. Notice the location of the horizontal and vertical asymptotes and compare these to the transformations)	→	→
Domain	x ≠ -1	x ≠ 2	x ≠ 2.5
Range	y ≠ 1	y ≠ 3	y ≠ .5
Asymptotes	x = -1 and y = 1	x = 2 and $y = 3$	x = 2.5 and $y = .5$

VIII. REVIEW of what I've already mastered: Perform the indicated operations, and simplify completely.

A)
$$\frac{x+3}{x-7} \cdot \frac{x^2-6x-7}{x^2-9}$$

B)
$$\frac{25x^2-100}{x^2-x-12} \div \frac{x^2-2x-24}{2x^2-72}$$

$$\frac{x+1}{x-3}$$

$$\frac{50(x+2)(x-2)(x+6)}{(x-4)(x+4)(x+3)} \text{ or } \frac{50x^3+300x^2-200x-1200}{(x-4)(x+4)(x+3)}$$

$$C) \quad \frac{\frac{1}{x+2}}{1+\frac{1}{x+2}}$$

D)
$$\frac{12 + \frac{1}{x} - \frac{1}{x^2}}{4 + \frac{1}{x}}$$

$$\frac{1}{x+3}$$

$$\frac{(4x-1)(3x+1)}{x(4x+1)}$$
 or $\frac{12x^2+x-1}{4x^2+x}$

E)
$$\frac{x}{x^2 - x - 12} + \frac{x - 2}{x^2 - 16}$$

F)
$$\frac{4}{x+6} - \frac{x+3}{x^2-36}$$

$$\frac{2x^2 + 5x - 6}{(x+4)(x-4)(x+3)}$$

$$\frac{3(x-9)}{x^2+x-2}$$

G)
$$\frac{x+a}{x-a} - \frac{x^2-a^2}{ax-a^2}$$

H)
$$\frac{3x+13}{x^2-3x-10} - \frac{16}{x^2-6x+5}$$

$$\frac{ax+2a^2-x^2}{a(x-a)}$$

$$\frac{3(x+9)}{x^2+x-2}$$