SECANTS AND TANGENTS

Objectives

After studying this section, you will be able to

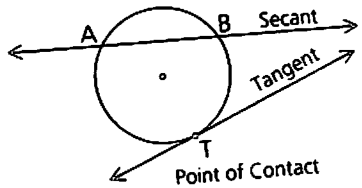
- Identify secant and tangent lines
- Identify secant and tangent segments
- Distinguish between two types of tangent circles
- Recognize common internal and common external tangents

Part One: Introduction

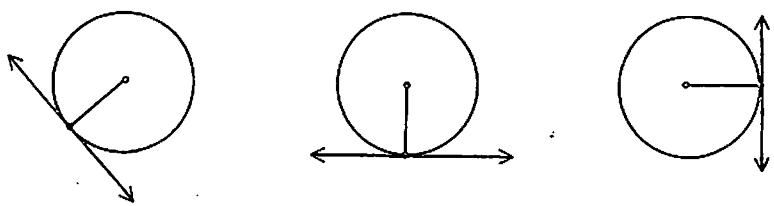
Secant and Tangent Lines

Some lines and circles have special relationships.

Definition A *secant* is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.)



Definition A *tangent* is a line that intersects a circle at exactly one point. This point is called the ***point of tangency*** or ***point of contact***.



The diagrams above suggest the following postulates about tangents.

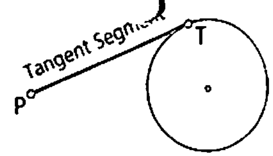
Postulate ***A tangent line is perpendicular to the radius drawn to the point of contact.***

Postulate ***If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.***

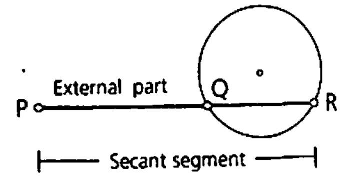
Secant and Tangent Segments

Some segments are related to circles in similar ways.

Definition A *tangent segment* is the part of a tangent line between the point of contact and a point outside the circle.



Definition A *secant segment* is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.

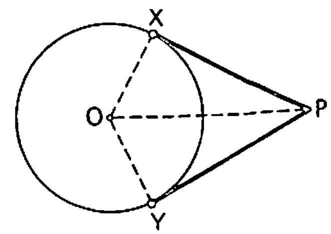


Definition The *external part* of a secant segment is the part of a secant line that joins the outside point to the nearer intersection point.

Theorem 85 ***If two tangent segments are drawn to a circle from an external point, then those segments are congruent. (Two-Tangent Theorem)***

Given: $\odot O$;
$\overline{PX}$ and $\overline{PY}$ are tangent segments.

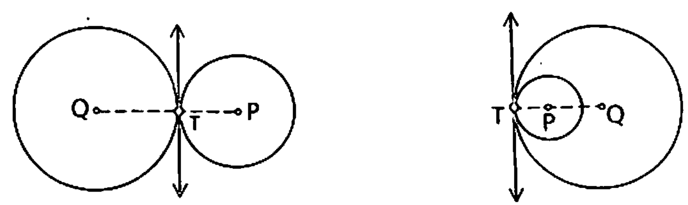
Prove: $\overline{PX} \cong \overline{PY}$



The Two-Tangent Theorem is easily proved with congruent triangles. More theorems relating to secant segments and tangent segments are presented in Section 10.8.

Tangent Circles

Definition *Tangent circles* are circles that intersect each other at exactly one point.



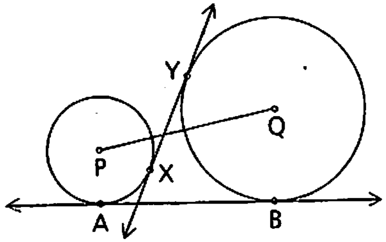
Definition Two circles are *externally tangent* if each of the tangent circles lies outside the other. (See the left-hand figure above.)

Definition Two circles are ***internally tangent*** if one of the tangent circles lies inside the other. (See the right-hand figure on the preceding page.)

Notice that in each case the tangent circles have one common tangent at their point of contact. Also, the point of contact lies on the ***line of centers***, $\overleftrightarrow{PQ}$.

Common Tangents

$\overleftrightarrow{PQ}$ is the line of centers.
$\overleftrightarrow{XY}$ is a ***common internal tangent.***
$\overleftrightarrow{AB}$ is a ***common external tangent.***



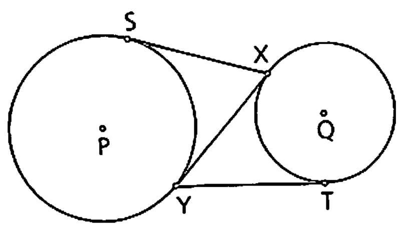
Definition A ***common tangent*** is a line tangent to two circles (not necessarily at the same point). Such a tangent is a ***common internal tangent*** if it lies between the circles (intersects the segment joining the centers) or a ***common external tangent*** if it is not between the circles (does not intersect the segment joining the centers).

In practice, we will frequently refer to a *segment* as a common tangent if it lies on a common tangent and its endpoints are the tangent's points of contact. In the preceding diagram, for example, $\overline{XY}$ can be called a common internal tangent and $\overline{AB}$ can be called a common external tangent.

Part Two: Sample Problems

Problem 1 Given: $\overline{XY}$ *is a common internal tangent to* ⊙ P *and* Q *at* X *and* Y.
$\overline{XS}$ *is tangent to* ⊙P *at* S.
$\overline{YT}$ *is tangent to* ⊙Q *at* T.

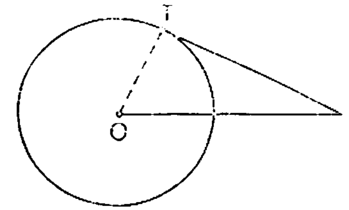
Conclusion: $\overline{XS} \cong \overline{YT}$



Proof

1 $\overline{XS}$ is tangent to ⊙P. $\overline{YT}$ is tangent to ⊙Q.	1 Given
2 $\overline{XY}$ is tangent to ⊙ P and Q.	2 Given
3 $\overline{XS} \cong \overline{XY}$ $\overline{XY} \cong \overline{YT}$	3 Two-Tangent Theorem 4 Same as 3
	5 Transitive Property

$\overleftrightarrow{TP}$ *is tangent to circle* O *at* T.
The radius of circle O *is* 8 *mm.*
Tangent segment $\overline{TP}$ *is* 6 *mm long.*
Find the length of $\overline{OP}$.

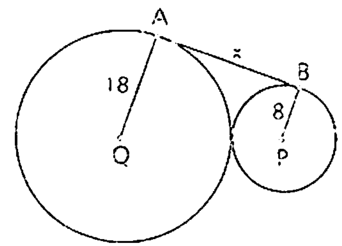


Draw radius $\overline{OT}$ to form right triangle OTP.

$$(TP)^2 + (TO)^2 = (OP)^2$$
$$6^2 + 8^2 = (OP)^2$$
$$\pm 10 = OP \qquad \text{(Reject } -10.)$$

Thus, OP = 10 mm.

A circle with a radius of 8 *cm is externally tangent to a circle with a radius of* 18 *cm. Find the length of a common external tangent.*



There is a standard procedure for solving a problem involving a common tangent (either internal or external).

Common-Tangent Procedure

1 Draw the segment joining the centers.

2 Draw the radii to the points of contact.

3 Through the center of the smaller circle, draw a line parallel to the common tangent.

4 Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle.

5 Use the Pythagorean Theorem and properties of a rectangle.

In △RPQ,

$$(QR)^2 + (RP)^2 = (PQ)^2$$
$$10^2 + (RP)^2 = 26^2$$
$$RP = \pm 24$$

Thus, AB = 24 cm.