

The properties of exponents can be used to solve exponential equations.

It sure would be nice if, as the first step, we could rewrite the equation so that the bases on both sides of the equation are the same. For **if** the bases on both sides of the equation are the same, **then** the exponents must be equal.

For instance, if $3^{x+1} = 9^x$

Both bases can be made the same... $3^{x+1} = (3^2)^x$

Using the exponent properties... $3^{x+1} = 3^{2x}$

If the bases are the same, then the exponents must be equal... $x+1 = 2x \Rightarrow \boxed{x = 1}$

DO NOT USE YOUR CALCULATOR as you do these problems:

1.) $2^x = 8$

$$\boxed{2^x = 2^3}$$

$$\boxed{x = 3}$$

2.) $27^{x+5} = 9^2$

$$(3^3)^{x+5} = (3^2)^2$$

$$3x + 15 = 4 \Rightarrow \boxed{x = -\frac{11}{3}}$$

3.) $5^{2x+3} = \frac{1}{125}$

$$5^{2x+3} = (5^{-3})$$

$$2x + 3 = -3$$

$$\boxed{x = -3}$$

4.) $\left(\frac{1}{2}\right)^{x+4} = 8^{2x-5}$

$$(2^{-1})^{x+4} = (2^3)^{2x-5}$$

$$-x - 4 = 6x - 15 \Rightarrow \boxed{x = \frac{11}{7}}$$

5.) $10000^{x+3} = 0.001^{x-3}$

$$(10^4)^{x+3} = (10^{-3})^{x-3}$$

$$4x + 12 = -3x + 9$$

$$\boxed{x = -\frac{3}{7}}$$

6.) $(0.2)^{2x-3} = 25^{\frac{x+6}{3}}$

$$(5^{-1})^{2x-3} = (5^2)^{\frac{x+6}{3}}$$

$$-2x + 3 = \frac{2x + 12}{3} \Rightarrow \boxed{x = -\frac{3}{8}}$$

7.) $\left(\frac{9}{16}\right)^{x^2-3} = \left(\frac{4}{3}\right)^{5-x^2}$

$$\left(\left(\frac{4}{3}\right)^{-2}\right)^{x^2-3} = \left(\frac{4}{3}\right)^{5-x^2}$$

$$-2x^2 + 6 = 5 - x^2$$

$$x^2 = 1 \Rightarrow \boxed{x = \pm 1}$$

8.) $\left(\frac{1}{25}\right)^{x-2} = 125^{5-x}$

$$(5^{-2})^{x-2} = (5^3)^{5-x}$$

$$-2x + 4 = 15 - 3x$$

$$\boxed{x = 11}$$

$$9.) \quad 2^{x^2+6} = 32^x$$

$$2^{x^2+6} = (2^5)^x$$

$$x^2 + 6 = 5x$$

$$x^2 - 5x + 6 = 0 \Rightarrow \boxed{x=2 \text{ or } x=3}$$

$$10.) \quad 9^{x^2-5} = 27^2$$

$$(3^2)^{x^2-5} = (3^3)^2$$

$$2x^2 - 10 = 6$$

$$2x^2 = 16 \Rightarrow \boxed{x = \pm 2\sqrt{2}}$$

$$11.) \quad \left(\frac{1}{2}\right)^{x+4} = 16^{x-1}$$

$$(2^{-1})^{x+4} = (2^4)^{x-1}$$

$$-x - 4 = 4x - 4$$

$$\boxed{x=0}$$

$$12.) \quad \left(\frac{1}{9}\right)^{x^2-3x} = 81^{5-x^2}$$

$$(3^{-2})^{x^2-3x} = (3^4)^{5-x^2}$$

$$-2x^2 + 6x = 20 - 4$$

$$2x^2 + 6x - 20 = 0 \Rightarrow \boxed{x = -5 \text{ or } x = 2}$$

$$13.) \quad 8^{7x} = 16^{3x+9}$$

$$(2^3)^{7x} = (2^4)^{3x+9}$$

$$21x = 12x + 36 \Rightarrow \boxed{x=4}$$

$$14.) \quad 2^{3x+5} = 7^{x-3}$$

Cannot be done, yet.

$$15.) \quad 27^{7x} = 81^{3x+9}$$

$$(3^3)^{7x} = (3^4)^{3x+9}$$

$$21x = 12x + 36$$

$$\boxed{x=4}$$

$$16.) \quad \left(\frac{1}{7}\right)^x = 49^{x+6}$$

$$(7^{-1})^x = (7^2)^{x+6}$$

$$-x = 2x + 12$$

$$\boxed{x=-4}$$

EXPONENT REVIEW: Simplify completely each of the following expressions:

$$17.) \quad \frac{-35x^4y^{-5}z^2}{14x^{-6}y^3z^3}$$

$$= \frac{-5x^{10}}{2y^8z}$$

$$18.) \quad \left(\frac{6m^{-4}n^3}{n^4r^{-2}}\right)^{-1}$$

$$= \frac{m^4n}{6r^2}$$

$$19.) \quad \left(\frac{1}{3}xy^2\right)(-3x^5y)^4$$

$$= 27x^{21}y^6$$

$$20.) \quad \left(\frac{x^a}{x^{2a-1}}\right)^{-4}$$

$$= x^{4a-4}$$

$$21.) \quad \frac{-3^4}{(-3)^5}$$

$$= \frac{1}{3}$$

$$22.) \quad [(-4x)^3(3x^5)^2]^2 = 331776x^{26}$$

$$23.) \quad \left(\frac{a^{3x-4}}{a^{x+1}}\right)^{-7} = a^{-14x+35} \text{ or } \frac{1}{a^{14x-35}}$$

$$24.) \quad \frac{3(2x^3y^2)^4}{(6x^{-5}y^3)^0} = 48x^{12}y^8$$